Series: **HYDRODYNAMICS**

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# The Emergence Mechanism and Calculation Method of Turbulent Flows

## Abstract

An explanation is offered of the mechanism of turbulent flows emergence, based of the Maxwell-like gravitation equations, updated after some known experiments. It is shown that the moving molecules of flowing liquid interact like electrical charges.

The forces of such interaction can be calculated and included to the Navier-Stokes equations as mass forces. Navier-Stocks equations complemented by these forces become equations of hydrodynamics for turbulent flow. For the calculations of turbulent flows the known methods of Navier-Stokes equations solution may be used.

## Contents



## 1. Introduction

In [1] an analogy of electromagnetism and gravito-electromagnetism has been explored. From the point of this analogy the new experiments of Samokhvalov [2] have been analyzed. Based on this analysis it was shown there, that Maxwell-like equations of gravito-electromagnetism should be supplemented by a certain empirical coefficient of gravitational permeability of the medium. For vacuum this coefficient is about

$\xi \approx 10^{12}$, and it decreases rapidly with the pressure increase. This explains the absence of visual effects of gravito-magnetic interaction of moving masses in the air. However in vacuum theses interactions are clearly visible in the above mentioned experiments.

In the liquid flow the moving molecules are separated by vacuum. So their gravito-magnetic interaction forces can be substantial and influence the nature of the flow.

We know that with increasing speed of laminar liquid or gas turbulence may occur spontaneously ( without the influence of external forces) [3]. The mechanism for turning from laminar to turbulent flow has not been found. Evidently, a source of forces perpendicular to the flow speed must be found.

Further it is shown that the gravito-magnetic interaction of the moving liquid masses can be the cause of the turbulence emergence.

## 2. The Interaction of Moving Electrical Charges

Let us consider two charges $q_1$ and $q_2$, moving with speeds $v_1$ and $v_2$ accordingly. It is known [4], that the induction of the field created by the charge $q_1$ in the point in which the charge $q_2$ is located, is equal (here and further we are using the CGS System)

$$\overline{B_1} = q_1\left(\overline{v_1} \times \overline{r}\right) / cr^3 . \tag{1}$$

Here the vector $\overline{r}$ is directed from the point where the moving charge $q_1$ is located. The Lorentz force acting on the charge $q_2$, is

$$\overline{F_{12}} = q_2\left(\overline{v_2} \times \overline{B_1}\right) / c . \tag{2}$$

Similarly,

$$\overline{B_2} = q_2\left(\overline{v_2} \times \overline{r}\right) / cr^3 , \tag{3}$$

$$\overline{F_{21}} = q_1\left(\overline{v_1} \times \overline{B_2}\right) / c . \tag{4}$$

In the general case $\overline{F_{12}} \neq \overline{F_{21}}$, i.e. the third Newton law does not work – there are unbalanced forces acting on the charges $q_1$ and $q_2$ and bending the trajectories of these charges movement.

Let us consider the correlation between the Lorentz force and the force of charges attraction. In the simplest case the Lorentz force found from (1, 2) is

$$F = \frac{q_1 q_2 v_1 v_2}{r^2 c^2}. \tag{5}$$

The force of attraction of the two charges is

$$P = \frac{q_1 q_2}{r^2}. \tag{6}$$

Consequently,

$$\phi_e = \frac{F}{P} = \frac{v_1 v_2}{c^2}. \tag{7}$$

We shall call this magnitude <u>the efficiency</u> of Lorentz forces.

## 3. Gravito-Magnetic Interaction of Moving Masses

In analogy with the electrical charges interaction, two masses $m_1$ and $m_2$, moving with speeds $v_1$ and $v_2$ accordingly are also interacting. In [1] it is shown that in this case there emerge gravito-magnetic inductions of the form:

$$\overline{B_{g1}} = G m_1 \left(\overline{v_1} \times \overline{r}\right) / c r^3, \tag{1}$$

$$\overline{B_{g2}} = G m_2 \left(\overline{v_2} \times \overline{r}\right) / c r^3, \tag{2}$$

where

$c$ — the speed of light in vacuum, $c \approx 3 \cdot 10^{10}$ [cm/sec];

$G$ - gravitational constant, $G \approx 7 \cdot 10^{-8}$ [dynes $\cdot$cm$^2$ $\cdot$g$^{-2}$].

The masses are also affected by the Lorentz gravito-magnetic Lorentz forces of the following form [1]:

$$\overline{F_{12}} = \varsigma \xi m_2 \left(\overline{v_2} \times \overline{B_{g1}}\right) / c, \tag{3}$$

$$\overline{F_{21}} = \varsigma \xi m_1 \left(\overline{v_1} \times \overline{B_{g2}}\right) / c, \tag{4}$$

where

$\varsigma = 2$, which follows from GRT,

$\xi \approx 10^{12}$ - <u>the gravitational permeability</u> coefficient for vacuum [1].

For parallel speeds and equal mass forces $\overline{F_{12}} = -\overline{F_{21}}$ the laminar flow keeps its character. However, in the general case, when $\overline{v_1} \neq \overline{v_2}$, the forces $\overline{F_{12}} \neq \overline{F_{21}}$ are generated, i.e. the unbalanced force

$\overline{\Delta F} = \overline{F_{12}} + \overline{F_{21}}$, acting on the masses $m_1$ and $m_2$ and bending the trajectories of these masses movement (let us note that here the Newton's third law is not observed [4]). From the above formulas follows that the unbalanced force is directed at an angle to the flow speed, which violates the laminarity.

Let us find the correlation between the Lorentz gravito-magnetic force and the masses attraction force. Similarly to the previous, in the simplest case Lorentz gravito-magnetic force may be found from (1, 3):

$$F = \varsigma\xi \frac{Gm_1 m_2 v_1 v_2}{r^2 c^2}. \quad (5)$$

The attraction force of two masses is

$$P = \frac{Gm_1 m_2}{r^2}. \quad (6)$$

Thus,

$$\phi_g = \frac{F}{P} = \varsigma\xi \cdot \frac{v_1 v_2}{c^2}. \quad (7)$$

We shall call this magnitude – the Lorentz gravito-magnetic force efficiency Comparing (2.7) and (3.7) we find that

$$\phi_g = \phi_e \varsigma\xi. \quad (8)$$

Consequently, the efficiency of Lorentz gravito-magnetic forces is much higher than Lorentz electromagnetic forces efficiency for comparable speeds.

## 4. Gravito-Magnetic Interaction as the Cause of Turbulence

For the appearance of unbalanced forces the following conditions must be satisfied:

1. the speeds must be of certain magnitude (which make these forces substantial);
2. there must be a cause for local change of the speeds, for instance
   - an appearance of a barrier,
   - a change of pressure when a stream flows from the water.

There may be a number of reasons increasing the unbalanced forces:

- Temperature rise causing the speeds $v_1$ and $v_2$ to cease being parallel due to heat fluctuations,

- Viscosity reduction, i.e. the reduction of intermolecular attraction forces which counteract the unbalanced forces that take the molecules apart.

We may specify also a number of external factors causing the appearance of unbalanced forces due to external violation of speeds $v_1$ and $v_2$ parallelism, for instance:

- abrupt changes in temperature, pressure;
- the injection of extra liquid or other agent.

A local change of equal speeds of a pair of linked molecules, caused, for instance, by asymmetric blow, is inevitably spread to the whole flow area.

As the Lorentz forces do not perform any work, the energy for turbulent motion must be supplied from the energy of laminar flow, which means that the energy of input flow must exceed a certain magnitude for the turbulence appearance.

The Navier-Stokes equations permit to determine of a speed of flow that meets a barrier or leaves a barrier. Knowing these speeds, we may determine unbalanced forces from the said equations. Then these forces as the functions of speeds, may be included into the Navier-Stokes equations as the mass forces.

## 5. Quantitative Estimates

In the general case, from (3.2, 3.4) we can find

$$\overline{F_{21}} = \frac{\varsigma\xi G m_1 m_2}{c^2 r^3}\left(\overline{v_1} \times \left(\overline{v_2} \times \overline{r}\right)\right). \tag{1}$$

Let us consider the vectors' orts, denoting them by a stroke. Then from (1) we get:

$$\overline{F_{21}} = \sigma \overline{f_{21}}, \tag{2}$$

where

$$\overline{f_{21}} = \left(\overline{v_1'} \times \left(\overline{v_2'} \times \overline{r'}\right)\right). \tag{3}$$

$$\sigma = \frac{\varsigma\xi G \cdot m_1 m_2 v_1 v_2}{c^2 r^2}, \tag{4}$$

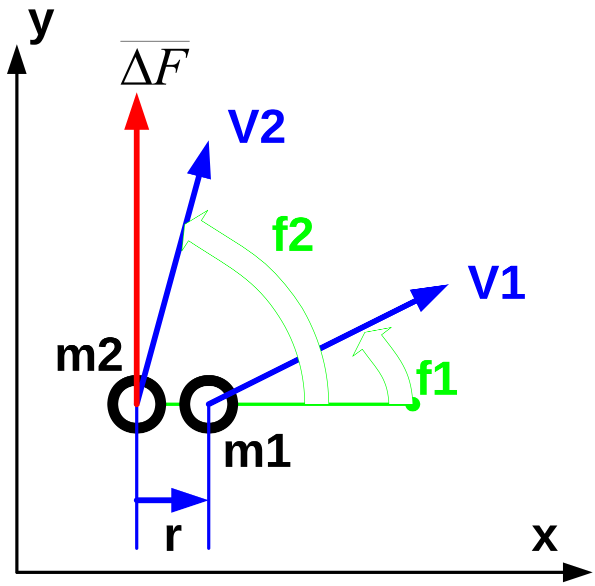


Fig. 1.

In the same way,

$$\overline{F_{12}} = \sigma \overline{f_{12}}, \tag{5}$$

where

$$\overline{f_{12}} = \left(\overline{v_2'} \times \left(\overline{v_1'} \times \overline{r'}\right)\right), \tag{6}$$

and

$$\overline{\Delta F} = \sigma \overline{\Delta f}, \tag{7}$$

where

$$\overline{\Delta F} = \overline{F_{21}} + \overline{F_{12}}, \tag{8}$$

$$\overline{\Delta f} = \overline{f_{21}} + \overline{f_{12}}. \tag{9}$$

Let us consider two adjacent molecules of the liquid. The distance between the molecules of liquid stays invariable. Due to the smallness of distance $r$ between them, we may assume that the vectors of speeds $\overline{v_1'}$, $\overline{v_2'}$ of these molecules are applied to one point and lie in the same plane $xoy$. Then vector (9) also lies in this plane. Fig. 1 shows the position of vectors $\overline{v_1'}$, $\overline{v_2'}$, $\overline{r'}$.

In the Supplement (see (6)) is shown that the magnitude of vector (9) is given by formula

$$\Delta f = r \sin(\varphi_2 - \varphi_1). \tag{8}$$

Taking into account (9, 10) we shall get:

$$\Delta F = \sigma \sin(\varphi_2 - \varphi_1). \qquad (9)$$

This force appears when the adjacent molecules hit the barrier under different angles. We may assume that the summary force is applied to one of the molecules. Thus it creates a torque of dipole consisting of two molecules,

$$M = r \cdot \Delta F. \qquad (10)$$

Each pair of adjacent molecules generates a dipole with a torque (10). The torques increase the local speeds of the liquid molecules, which, in its turn, increase the torques of the said dipoles. Because of this, the turbulence once began continues to grow, spreading in the liquid volume. Formula (9) determines the forces of gravito-magnetic interaction of the liquid molecules as a function of speeds of these contacting molecules.

These forces can be included to the Navier-Stokes equations as mass forces – see further.

## 6. Example: Turbulent Water Flow in a Pipe

Now we shall consider the case of interaction between liquid streams, assuming that the interaction is between groups of molecules forming an element of a stream. We shall take a specific case when the speed vectors of the streams are the same $|v_1| = |v_2| = v$ and also the masses of these groups are $m_1 = m_2 = m$. From (4) we can find

$$\sigma = \varsigma \xi G \left( \frac{mv}{cr} \right)^2. \qquad (11)$$

where $r$ is the distance between the streams. Let us denote as $d$ a typical size of the group (the stream diameter) and rewrite (11) as

$$\sigma = \varsigma \xi G \left( \frac{\rho \cdot d^3 v}{cr} \right)^2. \qquad (11a)$$

where $\rho$ is liquid density, and the mass of the group is

$$m = \rho \cdot d^3. \qquad (11в)$$

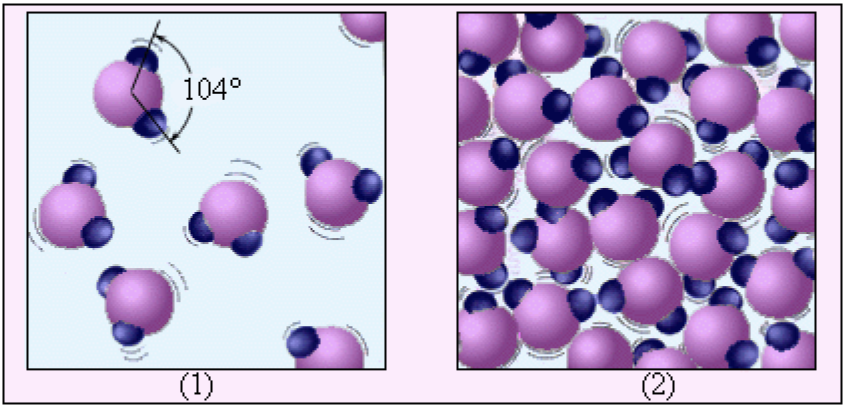


Fig. 2 (from Vikipedia). Water vapor (1) and water (2). Molecules of water are enlarged by $5 \cdot 10^7$ times.

The further example is related to water. As in liquids the molecules are located at a distance commensurable with the size of molecule itself (see Fig. 2), we shall take the distance between molecules equal to the molecule diameter, w2which for water is $r \approx 3 \cdot 10^{-12} [cm]$. The water density is $\rho = 1 \left[ g/cm^3 \right]$. Let us find also the speed of water flow where the turbulence occurs. It is known [3], that the condition of turbulence occurrence is given by Reinolds criterion, which for a round pipe is

$$\mathrm{Re} = Dv / \eta, \qquad (12)$$

where $D$ is the pipe's diameter, $\eta$ is the kinematic viscosity coefficient – for water it is $\eta \approx 0.01 \left[ cm^2/\sec \right]$ [5]. Let $D = 2.5 [cm]$. The turbulence occurs when the Reinolds number is $\mathrm{Re} > 2300$. Now from (12) let us find the speed of turbulent flow: $v = 10$ [cm/sec] Let the diameter of interacting streams $d \approx 0.1 [cm]$. It was mentioned above that $\varsigma = 2$, $\xi \approx 10^{12}$, $G \approx 7 \cdot 10^{-8}$. Then from (11a) we find

$$\sigma = 2 \cdot 10^{12} \cdot 7 \cdot 10^{-8} \left( 1 \cdot 0.1^3 \cdot 10 / \left( 3 \cdot 10^{10} \cdot 3 \cdot 10^{-12} \right) \right)^2 \approx 2000 \text{ [dynes]} \qquad (13)$$

Let us assume now that $\sin(\varphi_2 - \varphi_1) \approx 10^{-2}$. Then we shall find the force (9):

$$\Delta F \approx 20 \text{ [dynes]}. \qquad (14)$$

From (10, 14) we find torque:

$$M \approx r \cdot \Delta F \approx 2 \text{ [dynes*cm]}. \qquad (15)$$

# 7. The Equations of Turbulent Flow

Let us return to formula (5.1):

$$\overline{F_{21}} = \frac{\varsigma\xi G m^2}{c^2 r^3}\left(\overline{v_1}\times\left(\overline{v_2}\times\overline{r}\right)\right)\left[\mathrm{d}ynes = \frac{\mathrm{g}\cdot\mathrm{c}m}{sec^2}\right]. \qquad (1)$$

Similarly to p. 5 we find

$$\overline{\Delta F} = \vartheta\cdot\overline{\Delta f}\ , \qquad (2)$$

where

$$\vartheta = \frac{\varsigma\xi G m^2}{c^2 r^3}\left[\frac{g}{cm^2}\right], \qquad (3)$$

$$\overline{\Delta f} = \vartheta\left(\left(\overline{v_1}\times\left(\overline{v_2}\times\overline{r}\right)\right) - \left(\overline{v_2}\times\left(\overline{v_1}\times\overline{r}\right)\right)\right). \qquad (4)$$

Taking into account (11b), we rewrite (3) as

$$\vartheta = \frac{\varsigma\xi G \rho^2 d^6}{c^2 r^3}\left[\frac{g}{cm^2}\right]. \qquad (4a)$$

Further we shall denote the forces causing the turbulence, as T. In the Supplement is shown (see also Fig. 1), that if all forces lie in one plane, then (4) is equivalent to formula

$$T_y = \vartheta\cdot R_x\left(v_{2x}v_{1y} - v_{2y}v_{1x}\right), \qquad (5)$$

where

$T_y$ - is a force acting on the mass moving with speed $v_2$,

$R_x$ - the distance between the masses centers.

Let the two adjacent molecule groups are located on the $ox$ axis. We denote

$$R_x = dx, \qquad (6a)$$

$$v_2 = v,\ \ v_1 = v + dv. \qquad (6в)$$

Then

$$T_y = \vartheta\cdot dx\left(v_x\left(v_y + dv_y\right) - v_y\left(v_x + dv_x\right)\right) \qquad (7)$$

or

$$T_y = \vartheta\cdot dx\left(v_x dv_y - v_y dv_x\right). \qquad (8)$$

Similarly, for a right coordinate system we have:

$$T_z = \vartheta\cdot dy\left(v_y dv_z - v_z dv_y\right), \qquad (9)$$

$$T_x = \vartheta\cdot dz\left(v_z dv_x - v_x dv_z\right). \qquad (10)$$

Let us consider an operator (further for shortness sake we shall call it turbulean)

$$\Omega(v)=\begin{vmatrix} v_z \frac{dv_x}{dz} - v_x \frac{dv_z}{dz} \\ v_x \frac{dv_y}{dx} - v_y \frac{dv_x}{dx} \\ v_y \frac{dv_z}{dy} - v_z \frac{dv_y}{dy} \end{vmatrix} \left[\frac{см}{сек^2}\right]. \tag{11}$$

**Example 1.** We shall consider an ideal laminar flow in which $v_x \neq 0,\ v_y = 0,\ v_z = 0$. Apparently here $\Omega(v) = 0$, i.e. laminar flow cannot spontaneously become a turbulent flow.

According to (6a) we have

$$R = dx = dy = dz \tag{12}$$

From (10-12) follows the expression

$$T = R^2 \vartheta \cdot \Omega(v)\left[cm^2 \frac{g}{cm^2} \cdot \frac{cm}{sec^2} = \frac{g \cdot cm}{sec^2} = dynes\right]. \tag{13}$$

или

$$T = \vartheta_1 \cdot \Omega(v)[dynes], \tag{14}$$

где

$$\vartheta_1 = R^2 \vartheta = \frac{R^2 \varsigma \xi G \rho^2 d^6}{c^2 r^3}[g]. \tag{15}$$

The expression (14) defines a force acting on the group of molecules from the side of three adjacent molecule groups, located before the first group on the coordinate axes, if the differentials of the coordinates are equal to the distance between molecules (12). This force is acting on the volume of four molecule groups, i.e. on volume $4d^3$. So the force acting on a unit volume is

$$T_m = \rho_m \Omega(v)\left[\frac{dynes}{sm^3} = \frac{g}{sec^2 sm^2}\right], \tag{16}$$

where

$$\rho_m = \frac{\vartheta_1}{4d^3} = \frac{R^2 \varsigma \xi G \rho^2 d^3}{4c^2 r^3} \left[ \frac{\text{g}}{cm^3} \right]$$

or

$$\rho_m = \frac{\varsigma \xi G \rho^2 d^8}{4c^2 r^3} \left[ \frac{\text{g}}{cm^3} \right], \qquad (17)$$

поскольку $R \approx d$.

Note for comparison, that in hydrodynamics equations, the dimension of mass force is $F_m \left[ \frac{\text{dynes}}{\text{g}} = \frac{\text{cm}}{sec^2} \right]$, and the dimension of force acting on a unit volume. is $\rho F_m \left[ \frac{\text{dynes}}{\text{g}} \frac{\text{g}}{sm^3} = \frac{\text{dynes}}{sm^3} = \frac{\text{g}}{sec^2 \text{c}m^2} \right]$. The dimension of force (16) is exactly the same. The coefficient (17) has the dimension of density and it can be called the turbulent density of a liquid.

**Example 2.** Let us find the turbulent density $\rho_m$ of water. We have: $\rho = 1\left[\text{g/cm}^3\right]$, $d \approx 0.1[\text{c}m]$, $c \approx 3 \cdot 10^{10}[cm/\text{sec}]$, $\varsigma = 2$, $\xi \approx 10^{12}$. Let the diameter of the stream is $d \approx 0.1[\text{c}m]$ and the distance between the streams is $r \approx 10^{-8}[\text{c}m]$. Then

$$\rho_m = \frac{\varsigma \xi G \rho^2 d^8}{4c^2 r^3} = \frac{2 \cdot 10^{12} \cdot 7 \cdot 10^{-8} \cdot 10^{-8}}{4 \cdot \left(3 \cdot 10^{10}\right)^2 \left(10^{-8}\right)^3}$$

or $\rho_m \approx 0.4 \left[ \frac{\text{г}}{\text{см}^3} \right]$.

The forces (16) may be included into Navier-Stokes equations. The Navier-Stokes equations complemented by such forces become the equations of hydrodynamics for turbulent flow.

The turbulean (11) by its structure is similar to the expression

$$(v\cdot\nabla)v=\begin{bmatrix} v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z} \\ v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z} \\ v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z} \end{bmatrix}, \tag{18}$$

appearing in the Navier-Stokes equation. Thus for turbulent flows calculations one may use the known method for the Navier-Stokes equations solution, including the method presented in [6, 7].

The expression (18) arrears in Navier-Stokes solution with a factor $\rho$. Consequently, the turbulean (11) can influence the equation's solution if the coefficient (17) will be equal $\rho_m \approx \rho$.

## Supplement

Let us consider an expression with vectors

$$\overline{f}=\left(\overline{a}\times\left(\overline{b}\times\overline{r}\right)\right). \tag{1}$$

In a right Cartesian coordinate system this expression will look as follows:

$$\overline{f}=\begin{bmatrix} a_y\left(b_x r_y-b_y r_x\right)-a_z\left(b_z r_x-b_x r_z\right) \\ a_z\left(b_y r_z-b_z r_y\right)-a_x\left(b_x r_y-b_y r_x\right) \\ a_x\left(b_z r_x-b_x r_z\right)-a_y\left(b_y r_z-b_z r_y\right) \end{bmatrix}. \tag{2}$$

Let us assume that the projections of these vectors on the $z$ axis are equal to zero. Then

$$\overline{f}=\left(b_x r_y-b_y r_x\right)\begin{bmatrix} a_y \\ -a_x \\ 0 \end{bmatrix}. \tag{2a}$$

Let us also assume that $r_y=0$, i.e. $r=r_x$. Then

$$\overline{f} = rb_y \begin{bmatrix} -a_y \\ a_x \\ 0 \end{bmatrix}. \tag{3}$$

So, in the assumed conditions

$$\overline{f}_{ab} = \left(\overline{a} \times \left(\overline{b} \times \overline{r}\right)\right) = rb_y \begin{vmatrix} -a_y \\ a_x \end{vmatrix}. \tag{3a}$$

Similarly

$$\overline{f}_{ba} = \left(\overline{b} \times \left(\overline{a} \times \left(\overline{-r}\right)\right)\right) = -ra_y \begin{vmatrix} -b_y \\ b_x \end{vmatrix}.$$

We have

$$\overline{\Delta f} = \overline{f}_{ab} + \overline{f}_{ba} = r\left(\begin{vmatrix} 0 \\ a_x b_y - a_y b_x \end{vmatrix}\right) \tag{4}$$

or

$$\overline{\Delta f_y} = r\left(a_x b_y - a_y b_x\right) = rab\left(\cos\varphi_a \sin\varphi_b - \sin\varphi_a \cos\varphi_a\right), \tag{5}$$

where $\varphi_a$, $\varphi_b$ - the angles of vectors $a$, $b$ with the axis $ox$. Thus, vector $\overline{\Delta f}$ lies in the same plane where the initial vectors are located, this vector is directed along $oy$ axis and has the value

$$\Delta f = rab\sin\left(\varphi_b - \varphi_a\right). \tag{6}$$

## References


1. Khmelnik S.I. Experimental Clarification of Maxwell-similar Gravitation Equations. This issue.
2. Samokhvalov V.N. Papers in this issue.
3. Ivanov B.N. World of physical hydrodynamics. From the problems of turbulence to the physics of the cosmos. Ed. 2nd. - Moscow: Editorial URSS, 2010 (in Russian)
4. Zilberman G.E. Electricity and Magnetism, Moscow. "Science", 1970 (in Russian)
5. Wilner J.M. etc. Handbook of hydraulics and hydraulic drives, ed. "High School", 1976 (in Russian)

6. Khmelnik S.I. The existence and the search method for global solutions of Navier-Stokes equation. «The Papers of Independent Authors», Publisher «DNA», Israel, Printed in USA, Lulu Inc., catalogue 9748173, vol. 17, 2010, ISBN 978-0-557-88376-9
7. Khmelnik S.I. Navier-Stokes equations. On the existence and the search method for global solutions (second edition). Published by "MiC" - Mathematics in Computer Comp., printed in USA, printed in USA, Lulu Inc., ID 9976854, Israel, 2010, ISBN 978-1-4583-2400-9.